Symmetric Stability of Low Level Feature Detectors

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Abstract

We investigate the capability of low level feature detectors to consistently define feature keypoints in an image and its horizontally reflected (mirrored) image. It is our assertion that this consistency is a useful attribute of a feature detector and should be considered in assessing the robustness of a feature detector. We test ten of the most popular detectors using a popular dataset of 8,677 images. We define a set of error measurements to help us to understand the invariance in keypoint position, size and angle of orientation, and we use SIFT descriptors extracted from the keypoints to measure the consistency of extracted feature descriptors. We conclude that the FAST and CenSurE detectors are perfectly invariant to bilateral symmetry, *Good Features to Track* and the Harris Corner detector produce consistent keypoints that can be matched using feature descriptors, and others vary in their invariance. SIFT is the least invariant of all the detectors that we test.

1 Introduction

There are many feature detectors documented in the literature, and used in research and practical applications to fulfil the common need to identify interest points within an image. Information at these positions can then be extracted into a descriptor and used for correspondence matching in image retrieval and classification, image alignment, image stitching, and many other applications. The two stages are often combined into one in discussion, but each are independent and the algorithms used in each can often be interchanged.

Most popular and useful *feature descriptors* are invariant to scale and rotation, and matching features from two images where they appear at different sizes or are rotated can still be successful. Invariance to bilateral symmetry in *feature detectors*, however, is less well documented. We describe our interest in this invariance and investigate the property for some popular feature detectors, assessing their consistency in finding interest points within an image and a horizontal reflection of the image. Our goal is to identify which popular feature detectors are most invariant to bilateral symmetry, and what degree of error exists in the interest point position, size and orientation.

To the best of our knowledge, no assessment of low level feature detectors with respect to their invariance to bilateral symmetry has previously appeared in the literature. The main contributions in this paper are:

- we introduce five measurements of error that we show to be useful in determining the invariance to bilateral symmetry of a feature detector; *mean distance error*, *mean size error*, *mean angle error*, *mean descriptor distance error* and the *mean descriptor match error* (Section 4)
- we measure the accuracy of bilateral keypoint position, size and angle of orientation in an established dataset ([5]) of 8,677 images (Section 5)
- we evaluate the capability of popular detectors to find consistent interest points (Section 6)

2 Bilateral Symmetry

Bilateral Symmetry describes a symmetry through a vertical plane in an image, and can occur at different scales. Figure 1 shows two examples; (a) the image as a whole is bilaterally symmetrical because the right hand side of the plane (the dotted blue line down the centre of the image) is a mirror image of the left hand side and (b) the highlighted section of the image is bilaterally symmetrical although the image as a whole is not. Detected keypoints in an image are generally very small and detection of bilateral symmetry will be

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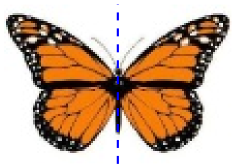
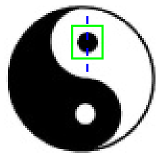
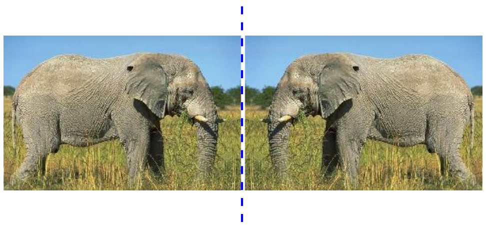


(a) Image bilateral symmetry



(b) Local bilateral symmetry



(c) Inter-image bilateral symmetry

Figure 1: Bilateral Symmetry (mirror reflection) at different scales

Figure 2: Motivating example. Cropped frames from a CCTV camera capture images of a man wearing a Nike hoodie (left) and later in the video having turned the hoodie inside-out, showing the Nike logo in reverse.

at a finer scale than both of these examples. Figure 1c shows our test case where we horizontally mirror the image to assess inter-image bilateral symmetry, and Figure 2 shows a real life example from a London street CCTV camera that demonstrates the need of reflection invariance in analysing CCTV images. A man wearing a hoodie exhibiting a Nike sportswear logo is later captured wearing the hoodie inside-out, with the Nike logo in reverse. Consistency in the detected position of a keypoint between an image and its horizontal reflection is important to enable a reflection invariant descriptor such as MIFT ([9]) to be extracted from the same point in the logo to maximise the potential to achieve a correspondence.

A keypoint is defined by its (x,y) co-ordinates, size, and sometimes, angle of orientation (Figure 3). We reflect a keypoint in its centre line (red dotted line). Let I be the image in which the keypoint is found, and I_x be the x-dimension of the image; the width. Let α be the angle of orientation, measured clockwise from $0°$parallel to the x-axis. Then, the new values for the x position of the keypoint is x' and the new angle of orientation is α',

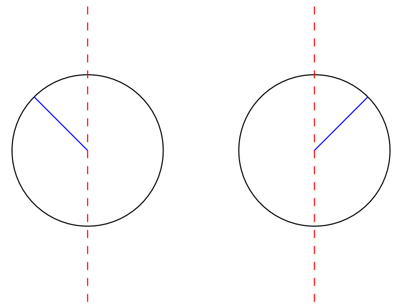


Figure 3: Reflecting a keypoint

thus

$$x' = I_x - x - 1 \tag{1}$$

$$\alpha' = \pi - \tan^{-1}\left(\frac{-sin\alpha}{-cos\alpha}\right) \tag{2}$$

3 Feature detectors

Feature detectors are used extensively in all areas of computer vision to identify parts of an image which contain pixel information that can be useful in many applications. Numerous detector methods have been described in the literature, and many have become popular for different tasks. Two distinct categories of feature detectors exist; *keypoint* detectors and *region detectors*. Recent trends in Deep Learning use features that are discovered automatically during the training process. In this paper, we concentrate on low-level feature detectors that can be described algorithmically, and how these perform in respect to reflection invariance.

[19] learn a ternary decision tree that can detect points with high repeatability, to create FAST; *Features from Accelerated Segment Test*. The BRISK detector ([12]) extends FAST with *an assembly of a bit-string* descriptor from intensity comparisons retrieved by dedicated sampling of each keypoint neighbourhood. ORB ([20]), is also based on the FAST detector from where the name is derived *Oriented FAST and Rotated BRIEF*, where BRIEF ([4]) is a feature descriptor. The Harris Corner detector (HARRIS, [10]) is a combined corner and edge detector based on the local auto-correlation function, and was extended by [21] to create *Good Features to Track* (GFTT). *Scale Invariant Feature Transform* (SIFT, [13]) is perhaps one of the most well known and commonly used detectors and uses a histogram of local oriented gradients that are measured in a pyramid of Gaussians to achieve scale invariance. *Speeded-Up Robust Features* (SURF, [2]) is a faster SIFT-inspired detector, using Hessian matrix to achieve good performance in computation time and accuracy. The final keypoint detector we evaluate is CenSurE ([1]), described as a fast variant of the upright SURF descriptor, and sometime called STAR.

In addition to keypoint detectors, we use two related region detectors from the same primary author; *Maximally Stable Extremal Regions* (MSER, [7]) for grey-scale images and *Maximally Stable Colour Regions* (MSCR, [6]).

4 Experiments and data

We assess the eight keypoint detectors and two region detectors described above, using the well established CALTECH101 dataset ([5]). The dataset consists of 8,677 JPEG images grouped into 101 categories, and contains a variety of image styles including cartoons and photographs of objects, human faces, animals and natural scenes. MSCR is the only detector that works with 3-channel colour images and for all other detectors, the original colour images are first converted to intensity images.

To measure the reflection invariance of the detectors with respect to bilateral symmetry, we use SIFT descriptors and measure their distance in feature space. Feature descriptors are themselves not invariance to bilateral symmetry and descriptors from an original image cannot be compared to a corresponding feature in a mirrored image. To overcome this, we extract feature descriptors from the original image using the detected keypoint attributes, and from reflected attributes detected in the mirror image. Let I represent an original image and M be the mirror of I. Then K_I and K_M represents keypoints detected in each of I and M respectively. Keypoints K_M are reflected as $k'_M \in K'_M$ using Equations (1) and (2), and feature descriptors are extracted from I using K'_M.

Our assessment is based on keypoint size and position. For features found by region detectors, we define a keypoint at the centre of the non-orthogonal (rotated) bounding rectangle of the region, and measure the size of the region as the encasing circle,

We ran experiments across the entire dataset, counting the number of keypoints found in each of the images from the dataset (the *original image*) and in the horizontally reflected image of the original (the *mirror image*). Horizontal reflection was performed using basic pixel swapping without interpolation to ensure that no artefacts were introduced into the image data. Statistics were collected for each detector. Keypoints were matched using brute-force exact matching, based only on their (x,y) position in each image.

For each image I and its mirror M, the difference in the number of keypoints found were tallied per detector. If more keypoints were found in I, then the difference is accumulated in D_1^d otherwise the difference is accumulated in D_2^d, where d denotes the detector. Values of D_1^d and D_2^d then demonstrate the variability in the detectors and their inability to find even a consistent number of keypoints in an image and its mirror.

Of course, counting the number of keypoints alone is not sufficient to measure quality, so we proceed to quantify the accuracy of the detected keypoints in position, scale and orientation. We first measure accuracy based upon keypoint position. Keypoints $k_I \in K_I$ and $k_M \in K_M$ are spatially matched to their nearest neighbour and the sub-pixel distance between each matched pair is accumulated and divided by the total number of keypoints to establish *mean distance error* for keypoints for each detector.

At each keypoint $k_I \in K_I$, and reflected keypoint $k'_M \in K'_M$, a SIFT descriptor is extracted from image I, giving S_I and S'_M respectively. These descriptors are matched using a brute-force L_2 distance matching algorithm in 128-dimensional space, and their distances accumulated to compute a mean distance error per detector. This *mean descriptor distance error* measures the average error found in matching a SIFT descriptor extracted from the original image at an original keypoint location, size and orientation, with a SIFT descriptor extracted from the original image using the keypoint attributes found in the mirror image, adjusted by horizontal reflection.

Our final metric of reflection invariance is a *mean descriptor match error*. We compare the results of matching keypoints $k_I \in K_I$ with $k'_M \in K'_M$ using two methods;

1. common descriptor matching based on the L_2 distance between the SIFT descriptors in 128-dimension feature space, and
2. spatial matching of keypoint position in the (x, y) image co-ordinate space.

We count the number of keypoints that do not correspond identically using the two methods and divide by the total number of matched keypoint pairs to give a mean value per descriptor. By comparing the results of the two keypoint matching strategies, we can determine an overall measure of how closely aligned the two sets of keypoints are, and therefore how robust the detector is to bilateral symmetry.

5 Results

Using the ten detectors, 41.78 million keypoints where found in the 8,677 images (Table 1). Overall, $1,330$ more keypoints were found in mirror images than in the original images, but this varied by descriptor. BRISK, SIFT, and MSER for example found 735, 644 and 437 more keypoints in the mirror images, but this represents only 0.05%, 0.02% and 0.06% increases. SURF found 494 fewer keypoints in the mirror image, 0.01%. Columns 4 and 5 of Table 1 show the number of category (of a total of 101) where fewer keypoints were found in the mirror image, or more keypoints were found in the mirror image. There was no consistency in the variance of the number of keypoints found to the category with each detector used.

Table 1: Breakdown of keypoint metrics per detector, across all 8,677 images in the CALTECH101 dataset. From left to right; (a) Detector name, (b) Number of keypoints found in all the original images, and in the mirror images (c), (d) Number of keypoints matched between the images and their mirror image, (e) Number of categories with fewer keypoints found in the mirror images, (f) Number of categories with more keypoints found in the mirror images, (g,h) tally of keypoints found in each set of images and not the other, see text for details

Detector	Number of keypoints in the original images	Number of keypoints in mirror images	Number of matches	Excess keypoints in original images D_1^d	Excess keypoints in mirror images D_2^d
BRISK	1,443,461	**1,444,196**	1,401,422	16,409	17,144
FAST	17,013,915	17,013,915	17,013,915	0	0
GFTT	**6,865,916**	6,865,915	6,865,887	15	14
HARRIS	3,361,261	**3,361,262**	3,361,257	1	2
ORB	3,566,926	**3,566,931**	3,566,797	5	10
SIFT	3,066,878	**3,067,522**	3,005,498	20,858	21,502
CenSurE	395,124	395,124	395,124	0	0
SURF	**4,664,275**	4,663,781	4,641,957	8,474	7,980
MSCR	663,284	**663,287**	662,871	304	307
MSER	740,002	**740,439**	722,555	6,682	7,119
	41,781,042	41,782,372	41,637,283		

Table 2: Measurements of accuracy in the spatial domain

Detector	Mean Distance Error	Mean Size Error	Mean Angle Error
BRISK	1.98	0.434	9.28
FAST	**0.00**	**0.000**	–
GFTT	**8.07E-06**	**0.000**	–
HARRIS	**4.19E-07**	**0.000**	–
ORB	0.48	1.324	2.61
SIFT	2.10	0.214	11.52
CenSurE	**0.00**	**0.000**	–
SURF	0.22	0.125	0.76
MSCR	0.01	0.005	0.22
MSER	0.86	0.317	1.49

Two detectors appear to perform well in handling bilateral symmetry of an image and its mirror. The FAST and CenSurE detectors both find an identical number of keypoints in every image, across all categories of the 8,677 image dataset. Both also have a zero *mean distance error* (Table 2 column 2) indicating an exact match. GFTT and HARRIS have very small errors $< 10^{-6}$. The same protocol is followed to measure the mean error in the size of the keypoint (column 3) and the mean error in the angle of orientation of the keypoint (column 4). Four of the keypoint detectors do not define an angle of orientations; FAST, GFTT, HARRIS and CenSurE. For these detectors, there is no *mean angle error*.

In a keypoint detector that is perfectly invariant to bilateral symmetry, the *mean descriptor distance error* (Table 3, column 2) value is 0.0, as is the case for four of the tested descriptors; FAST, GFTT, HARRIS and CenSurE. However, *mean descriptor distance error* of 0.0 alone cannot determine perfect invariance to bilateral symmetry. Our observations in Table 1 tell us that the GFTT and HARRIS feature detectors produce a different number of keypoints in I and M, so while the descriptors can be matched with zero error, the spatial position of matched keypoints k_I and k_M have not been proven to be consistent.

The *mean descriptor match error* for each detector is shown in Table 3, column 3. The four detectors FAST, GFTT, HARRIS and CenSurE all show a 0.0 match error, demonstrating that keypoints $k_M \in K_M$ found by these detectors can be matched to identical keypoints in $k_I \in K_I$ in the original images.

6 Discussion

6.1 Error measurements

The *mean descriptor distance error* and *mean descriptor match error* are perhaps the most important measurements in assessing invariance to bilateral symmetry. *Mean descriptor distance error* is the average Euclidean distance between matched descriptors measured in 128-dimension descriptor space. A mean of 0.0 indicates perfect matching. *Mean descriptor match error* is a measure of the matching accuracy based on descriptors against matching spatially. In a perfect set of bilateral feature keypoints, the feature descriptor match would yield the same keypoint pairings as matching keypoints spatially.

Eight out of ten feature detectors that we tested found a different number of keypoints in an image and in the mirror of the image. FAST and CenSurE detectors were the two exceptions. The initial test identified that these two detectors were consistent in the number of keypoints that they were able to detect and further experiments confirmed that there was consistency across all 101 categories. Our measures of error – *mean distance error*, *mean size error*, *mean descriptor distance error* and *mean descriptor match error* – all confirmed perfect bilateral symmetry in all of the $17,013,915$ and $395,124$ keypoints, respectively. It is important to note, though, that these detectors only determine location and size of a feature, and do not define the angle of orientation of the feature, which we conclude to be a significant factor in its invariance. The fifth error measure, *mean angle error* is therefore omitted for these detectors. Nonetheless, the invariance in an important attribute of the detectors for location and size.

Other detectors that do not identify the angle of orientation of the feature keypoint – GFTT and HARRIS – also performed well in our error measurement tests. The number of keypoints detected in the images and their mirror reflected images varied in 18 and 3 categories respectively and the detectors therefore cannot be seen as perfectly invariant to bilateral symmetry. However, all the error measurements are 0.0, except for the *mean distance error* which are $8.07E-06$ and $4.19E-07$ respectively. With such small errors across $6,865,916$ and $3,361,261$ feature keypoints, it is fair to conclude these to also be invariant to bilateral symmetry, given the *mean descriptor distance error* and *mean descriptor match error* are both 0.0. It should be noted though, that feature matching and filtering is required to achieve correct correspondence between non-identical sets of keypoint features.

The region-based detectors MSCR and MSER generally identify fewer features – 663,284 and 740,002 in the original images – and demonstrated a greater invariance to bilateral symmetry than keypoint detectors with orientation. MSCR feature positions are very consistent, with a *mean distance error* of 0.01 pixels and *mean size error* of 0.005 pixels and the *mean angle error* is only $0.22°$. Corresponding MSER error values are 0.86 pixels, 0.317 pixels and $1.49°$. MSCR (on colour images) has a low *mean descriptor match error* of 0.88. MSER using the same underlying algorithm on grey-scale images has a larger error of 5.92 suggesting that colour helps with invariance to bilateral symmetry in the maximally stable region algorithm.

The final four detectors have much larger error measurements in some or all of our position, size and angle metrics (Table 2), which cause some large error values in the descriptors (Table 3). SURF has the lowest *mean descriptor match error* of 7.32 and BRISK, ORB and SIFT show much higher error values of 26.81, 39.38 and 62.46 respectively. In an image and mirror image pair, correspondence can be expected to fail with this accuracy on this number of keypoint matches.

6.2 Possible causes for reflection invariance

The FAST detector analyses the set of pixels in close proximity to a candidate pixel and classifies each pixel as a corner or non-corner pixel without regard of the relative spatial layout of the neighbourhood. The BRISK derivative introduces sampling of the pixel neighbourhood and consequently loses the inherent reflection invariance. ORB, another FAST derivative, uses an intensity centroid ([18]) to measure orientation, which may suggest a cause for the error that we have observed. HARRIS is invariant through its use of local auto-correlation, and GFTT largely maintains this property in its extension. SIFT sub-samples the image at higher scales, losing pixel-precision as [1] observes, and this pixel-level imprecision is further observed in our experiments. SURF's use of a Hessian matrix improves on SIFT's invariance. The two region detectors look for stable regions, similar to a watershed algorithm. The use of three colour channels in MSCR shows a large improvement in the region stability in the oriented image.

Table 3: Measurements of accuracy based on SIFT feature descriptors

Detector	Mean Descriptor Distance Error	Mean Descriptor Match Error
BRISK	58.02	26.81
FAST	**0.00**	**0.00**
GFTT	**0.00**	**0.00**
HARRIS	**0.00**	**0.00**
ORB	30.53	39.38
SIFT	113.32	62.46
CenSurE	**0.00**	**0.00**
SURF	3.65	7.32
MSCR	0.70	0.88
MSER	13.68	5.92

We observe that design and implementation choices both contribute to invariance to bilateral symmetry in common feature detectors. Scale-invariant detectors, for example, smooth pixel values when scaling an image, which introduces pixel value changes sensitive to surrounding values and leads to invariance. This is a design issue.

Other invariance is caused by implementation choices. For example, algorithms that use a Difference-of-Gaussian pyramid (such as SIFT) for sub-pixel feature detection can inadvertently increase their reflection dependence by using 32-bit floating point arithmetic for intermediate calculations. Using the popular OpenCV ([3]) library – version $2.4.12$ for C++ – we tested the `GaussianBlur()` function that convolves an image with a specified Gaussian kernel. We found that the library implementation that uses 32-bit floating-point arithmetic produces reflection-sensitive convolutions for many images. We re-implemented the algorithm using 64-bit floating-point arithmetic and all convolutions of our test images were reflection invariant. This demonstrates that the implementation choice of using 32-bit floating-point arithmetic introduces a rounding error which can subsequent cause invariance in the dependent interest point calculations.

Table 4: Conclusions of the invariance characteristics of ten feature detectors use in our experiments

Detector	Invariant
BRISK	No
FAST	**Perfect**
GFTT	Yes, after matching
HARRIS	Yes, after matching
ORB	No
SIFT	No
CenSurE	**Perfect**
SURF	No
MSCR	Somewhat
MSER	Somewhat

7 Related work

Most work on bilateral symmetry concentrates on the detection of symmetry and accurate positioning of the line of symmetry within a single image (e.g. [16, 17], and [22]; see [14] for background work on Symmetry). [24] investigated object localization methods and concluded that all of the methods that they evaluated on two representative problems struggle to get mirror symmetric results. The authors introduced the concept of *mirrorability* in their assessment of accurate face alignment and human pose estimation to qualitatively analyse method behaviours. Many reflection invariant feature descriptors have been proposed, but none have corresponding reflection invariant detectors; MIFT ([9]), MI-SIFT ([15]), F-SIFT ([26]), FIND ([8]), FIS ([25]), mirror reflection invariant HOG descriptors ([11]), and Max-SIFT ([23]).

8 Conclusion

We have assessed ten popular image feature detectors to determine their invariance to bilateral symmetry. We focussed on the accuracy and consistency of feature detection between an image and its mirror reflection. We conclude (Table 4) that FAST and CenSurE detectors are perfectly invariant and GFTT and the Harris Corner detector are invariant after feature matching and filtering algorithms are applied to find the correct correspondences in uneven sized sets of detected interest points. BRISK, ORB, SIFT and SURF cannot be considered invariant to bilateral symmetry, and SIFT is the least invariant of all the detectors that we have experimented with. Region-based detectors MSCR and MSER were also assessed based on a common approach of defining a keypoint at the centre of the detected region. In this case, MSCR is largely invariant and MSER is somewhat invariant, indicating that colour plays an important role in the invariance of maximally stable region algorithm.

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